

分段数列专项参考答案:

1. 解析因为 $a_1=1$, $a_n+a_{n+1}=3$,

$$\text{所以 } S_{2023} = a_1 + (a_2 + a_3) + (a_4 + a_5) + \cdots + (a_{2022} + a_{2023}) = 1 + 3 \times \frac{2023-1}{2} = 3034.$$

2. 解析由 $a_{n+1} + a_n = 3 \cdot 2^n$, 得 $a_1 + a_2 = 6$, 而 $a_2 = 5$, 解得 $a_1 = 1$,

所以 $\{a_n\}$ 的前 11 项的和 $a_1 + (a_2 + a_3) + (a_4 + a_5) + (a_6 + a_7) + (a_8 + a_9) + (a_{10} + a_{11})$

$$= 1 + 3(2^2 + 2^4 + 2^6 + 2^8 + 2^{10}) = 1 + 3(4 + 16 + 64 + 256 + 1024) = 4093.$$

$$S_{31} = a_1 + (a_2 + a_3) + (a_4 + a_5) + \cdots + (a_{30} + a_{31}) = 1 + 2 \times 2 + 2 \times 4 + \cdots + 2 \times 30$$

$$= 1 + 2 \times (2 + 4 + \cdots + 30) = 1 + 2 \times \frac{15 \times (2 + 30)}{2} = 481.$$

3. 解析因为 $a_1 = -1$, $a_2 = -3$, $a_n a_{n+2} = -3$,

所以 $a_3 = 3$, $a_4 = 1$, $a_5 = -1$, $a_6 = -3$, $a_7 = 3$, $a_8 = 1$, \cdots ,

所以 $\{a_n\}$ 是以 4 为周期的周期数列, 且 $a_1 + a_2 + a_3 + a_4 = 0$,

$$a_{2021} = a_1 = -1, \quad a_{2022} = a_2 = -3, \quad a_{2023} = a_3 = 3,$$

$$\text{所以 } S_{2023} = 505(a_1 + a_2 + a_3 + a_4) + a_1 + a_2 + a_3 = -1.$$

$$4. \text{ 解析由题意知 } a_n = \begin{cases} \frac{n^2-1}{2}, n \text{ 为奇数} \\ \frac{n^2}{2}, n \text{ 为偶数} \end{cases}, \quad b_n = (-1)^n a_n,$$

故数列 $\{b_n\}$ 的前 30 项和为 $-a_1 + a_2 - a_3 + a_4 - \cdots - a_{29} + a_{30}$

$$= -\frac{1^2-1}{2} + \frac{2^2}{2} - \frac{3^2-1}{2} + \frac{4^2}{2} - \cdots - \frac{29^2-1}{2} + \frac{30^2}{2}$$

$$= \frac{2^2-1^2}{2} + \frac{4^2-3^2}{2} + \cdots + \frac{30^2-29^2}{2} + \frac{15}{2}$$

$$= \frac{1}{2}(3+7+11+\cdots+59) + \frac{15}{2}$$

$$= \frac{1}{2} \times \frac{15(3+59)}{2} + \frac{15}{2} = 240,$$

5. 解析因为 $a_{2n} = a_{2n-1} + 1, a_{2n+1} = a_{2n} + 2$, 所以 $a_{2n+1} = a_{2n} + 2 = a_{2n-1} + 3$,

即 $a_{2n+1} - a_{2n-1} = 3$, 所以数列 $\{a_n\}$ 的奇数项是以 1 为首项, 3 为公差的等差数列;

同理, 由 $a_{2n+2} - a_{2n} = 3$ 知,

数列 $\{a_n\}$ 的偶数项是以 2 为首项, 3 为公差的等差数列.

从而数列 $\{a_n\}$ 的前 20 项和为:

$$S_{20} = (a_1 + a_3 + a_5 + \cdots + a_{19}) + (a_2 + a_4 + a_6 + \cdots + a_{20}) = 10 \times 1 + \frac{10 \times 9}{2} \times 3 + 10 \times 2 + \frac{10 \times 9}{2} \times 3 = 300.$$

6. 设等差数列 $\{a_n\}$ 的公差为 d , 而 $b_n = \begin{cases} a_n - 6, & n = 2k - 1 \\ 2a_n, & n = 2k \end{cases}, k \in \mathbf{N}^*$,

$$\text{则 } b_1 = a_1 - 6, b_2 = 2a_2 = 2a_1 + 2d, b_3 = a_3 - 6 = a_1 + 2d - 6,$$

$$\text{于是 } \begin{cases} S_4 = 4a_1 + 6d = 32 \\ T_3 = 4a_1 + 4d - 12 = 16 \end{cases}, \text{ 解得 } a_1 = 5, d = 2,$$

则 $a_n = a_1 + (n-1)d = 2n + 3$, 所以数列 $\{a_n\}$ 的通项公式是 $a_n = 2n + 3$,

$$\text{则 } S_n = \frac{n(5 + 2n + 3)}{2} = n^2 + 4n, \quad b_n = \begin{cases} 2n - 3 & n = 2k - 1 \\ 4n + 6 & n = 2k \end{cases}, \quad k \in \mathbf{N}^*,$$

当 n 为偶数时, $b_{n-1} + b_n = 2(n-1) - 3 + 4n + 6 = 6n + 1$,

$$\text{所以 } T_n = \frac{13 + 6n + 1}{2} \times \frac{n}{2} = \frac{3}{2}n^2 + \frac{7}{2}n,$$

$$\text{当 } n \text{ 为奇数时, } T_n = T_{n+1} - b_{n+1} = \frac{3}{2}(n+1)^2 + \frac{7}{2}(n+1) - [4(n+1) + 6] = \frac{3}{2}n^2 + \frac{5}{2}n - 5,$$

$$\text{所以 } T_n = \begin{cases} \frac{3}{2}n^2 + \frac{7}{2}n, & n = 2k \\ \frac{3}{2}n^2 + \frac{5}{2}n - 5, & n = 2k - 1 \end{cases}, \quad k \in \mathbf{N}^*.$$

$$\text{故答案为: } T_n = \begin{cases} \frac{3}{2}n^2 + \frac{7}{2}n, & n = 2k \\ \frac{3}{2}n^2 + \frac{5}{2}n - 5, & n = 2k - 1 \end{cases}, (k \in \mathbf{N}^*).$$

7. 解析若 $n = 2k - 1, k \in \mathbb{N}^*$, 则 $a_{2k+1} = a_{2k-1} + 1 + (-1)^{2k-1} = a_{2k-1}$,

若 $n = 2k, k \in \mathbb{N}^*$, 则 $a_{2k+2} = a_{2k} + 1 + (-1)^{2k} = a_{2k} + 2$,

所以数列 $\{a_n\}$ 的偶数项构成以 2 为首项, 公差为 2 的等差数列, 奇数项构成常数数列,

$$S_{100} = 50 \times 1 + \left(50 \times 2 + \frac{50 \times 49}{2} \times 2 \right) = 2600.$$

8. 【解析】解: 由 $a_1 = 1$, $a_{2n} = a_{2n-1} + (-1)^n$, $a_{2n+1} = a_{2n} + 3^n (n \in \mathbb{N}^*)$, 得

$$a_{2n+1} = a_{2n} + 3^n = a_{2n-1} + (-1)^n + 3^n,$$

$$a_{2n-1} = a_{2n-3} + (-1)^{n-1} + 3^{n-1},$$

$$a_{2n-3} = a_{2n-5} + (-1)^{n-2} + 3^{n-2},$$

...

$$a_5 = a_3 + (-1)^2 + 3^2,$$

$$a_3 = a_1 + (-1)^1 + 3^1,$$

累加得: $a_{2n-1} + a_{2n-3} + \dots + a_5 + a_3 = a_{2n-3} + a_{2n-5} + \dots + a_3 + a_1$

$$+ (-1)^1 + (-1)^2 + \dots + (-1)^{n-2} + (-1)^{n-1} + 3^1 + 3^2 + \dots + 3^{n-2} + 3^{n-1},$$

$$\therefore a_{2n-1} = a_1 + \frac{-1 \times [1 - (-1)^{n-1}]}{2} + \frac{3 \times (1 - 3^{n-1})}{1 - 3}$$

$$= 1 - \frac{1}{2} + \frac{1}{2} \times (-1)^{n-1} + \frac{3}{2} \times 3^{n-1} - \frac{3}{2} = \frac{1}{2} \times (-1)^{n-1} + \frac{3}{2} \times 3^{n-1} - 1.$$

$$\therefore a_{2n} = a_{2n-1} + (-1)^n = \frac{3}{2} \times 3^{n-1} - \frac{1}{2} \times (-1)^{n-1} - 1.$$

则 $S_{2017} = (a_1 + a_3 + a_5 + \dots + a_{2017}) + (a_2 + a_4 + a_6 + \dots + a_{2016})$

$$= \frac{1}{2} [(-1)^0 + 3^1 - 1 + (-1)^1 + 3^2 - 1 + \dots + (-1)^{1008} + 3^{1009} - 1]$$

$$+ \frac{1}{2} [3^1 - (-1)^0 - 1 + 3^2 - (-1)^1 - 1 + \dots + 3^{1008} - (-1)^{1007} - 1]$$

$$= 3 + 3^2 + 3^3 + \dots + 3^{1008} + \frac{1}{2} \times 3^{1009} - 2016 - \frac{1}{2}$$

$$= \frac{3 \times (1 - 3^{1008})}{1 - 3} + \frac{1}{2} \times 3^{1009} - 2016 - \frac{1}{2}$$

$$= 3^{1009} - 2018.$$

故选: D.

9. 【解析】解：根据题意，数列 $\{a_n\}$ 满足 $a_{n+1} + a_n = (-1)^n(2n-1)$ ，当 n 为奇数时，有

$$a_{n+1} + a_n = -(2n-1),$$

其中当 $n=1$ 时，有 $a_2 + a_1 = -1$ ，

当 $n=3$ 时，有 $a_4 + a_3 = -5$ ，

当 $n=5$ 时，有 $a_6 + a_5 = -9$ ，

...

当 $n=59$ 时，有 $a_{60} + a_{59} = -(2 \times 59 - 1) = -117$ ，

则 $\{a_n\}$ 的前 60 项和 $S_{60} = (a_2 + a_1) + (a_4 + a_3) + \dots + (a_{60} + a_{59})$

$$= (-1) + (-5) + \dots + (-117) = -(1 + 5 + 9 + \dots + 117)$$

$$= -\frac{(1+117) \times 30}{2} = -1770;$$

故选：C.

10. 【解析】解：数列 $\{a_n\}$ 满足 $a_{n+1} + (-1)^n a_n = 2n-1$ ， $a_1 = 1$ ， $\therefore a_2 - 1 = 1$ ，解得 $a_2 = 2$ 。

$\therefore a_3 + 2 = 3$ ，解得 $a_3 = 1$ 。

$$\therefore a_{n+1} + (-1)^n a_n = 2n-1,$$

\therefore 有 $a_2 - a_1 = 1$ ， $a_3 + a_2 = 3$ ， $a_4 - a_3 = 5$ ， $a_5 + a_4 = 7$ ， $a_6 - a_5 = 9$ ， $a_7 + a_6 = 11$ ，

$$\dots a_{50} - a_{49} = 97.$$

从而可得 $a_3 + a_1 = 2$ ， $a_4 + a_2 = 8$ ， $a_7 + a_5 = 2$ ， $a_8 + a_6 = 24$ ， $a_9 + a_{11} = 2$ ， $a_{12} + a_{10} = 40$ ，

$$a_{13} + a_{11} = 2，a_{16} + a_{14} = 56，\dots$$

从第一项开始，依次取 2 个相邻奇数项的和都等于 2，从第二项开始，依次取 2 个相邻偶数项的和构成以 8 为首项，以 16 为公差的等差数列。

$$\therefore \{a_n\} \text{ 的前 60 项和为 } 15 \times 2 + (15 \times 8 + \frac{15 \times 14}{2}) = 1830,$$

故答案为：1，1830.

11. 【解析】解：数列 $\{a_n\}$ 满足 $a_1 = 1$ ， $a_{n+1} \cdot a_n = 2^n (n \in N^*)$ ，

当 $n=1$ 时，解得 $a_2 = 2$ ，

所以 $\frac{a_{n+2}}{a_n} = 2$ （常数），

所以数列 a_1, a_3, a_5, \dots 是以 1 为首项，2 为公比的等比数列，

同理数列 a_2, a_4, a_6, \dots 是以 2 为首项，2 为公比的等比数列。

$$\text{所以 } S_{2016} = (a_1 + a_3 + \dots + a_{2015}) + (a_2 + a_4 + \dots + a_{2016}) = \frac{(2^{1008} - 1)}{2 - 1} + \frac{2 \times (2^{1008} - 1)}{2 - 1} = 3 \times 2^{1008} - 3.$$

故选：B.

12 【解析】解：当 n 是奇数时， $\cos n\pi = -1$ ；当 n 是偶数时， $\cos n\pi = 1$.

$$\text{则 } a_n = (-1)^n (n^2 + 4n) = (-1)^n n^2 + (-1)^n \times 4n,$$

$$\{a_n\} \text{ 的前 } 50 \text{ 项的和 } S_{50} = a_1 + a_2 + a_3 + \dots + a_{50},$$

$$= (-1^2 + 2^2 - 3^2 + 4^2 - \dots + 50^2) + 4(-1 + 2 - 3 + 4 - \dots + 50),$$

$$= (1 + 2 + 3 + 4 + \dots + 50) + 4 \times 25,$$

$$= 1275 + 100,$$

$$= 1375,$$

故答案为：1375

13. 【解析】解：函数 $f(n) = n^2 \cos(n\pi)$ ，数列 $\{a_n\}$ 满足 $a_n = f(n) + f(n+1) (n \in N^+)$ ，

$$a_{2k-1} = f(2k-1) + f(2k) = -(2k-1)^2 + (2k)^2 = 4k - 1.$$

$$a_{2k} = f(2k) + f(2k+1) = (2k)^2 - (2k+1)^2 = -4k - 1.$$

$$\therefore a_{2k-1} + a_{2k} = -2.$$

$$\therefore a_1 + a_2 + \dots + a_{2n} = -2n.$$

故答案为： $-2n$.

14 【解析】解：数列 $\{a_n\}$ 满足 $a_{n+1} = (2|\sin \frac{n\pi}{2}| - 1)a_n + 2n, n \in N^*$ ，

$$n = 2k (k \in N^*) \text{ 为偶数时， } a_{n+1} = -a_n + 2n, \text{ 即 } a_{2k+1} + a_{2k} = 4k.$$

$$n = 2k - 1 (k \in N^*) \text{ 为奇数时， } a_{n+1} = a_n + 2n, \text{ 即 } a_{2k} - a_{2k-1} = 4k - 2.$$

$$\text{相减可得： } a_{2k+1} + a_{2k-1} = 2.$$

$$\text{由 } a_{2k} - a_{2k-1} = 4k - 2, \text{ 可得： } a_{2k+2} - a_{2k+1} = 4k + 2.$$

$$\text{可得： } a_{2k+2} + a_{2k} = 8k + 2.$$

$$\text{则数列 } \{a_n\} \text{ 的前 } 60 \text{ 项和} = (a_1 + a_3 + \dots + a_{59}) + (a_2 + a_4 + \dots + a_{60})$$

$$= 30 + [(8+2) + (8 \times 3 + 2) + \dots + (8 \times 29 + 2)]$$

$$= 30 + \frac{15 \times (10 + 234)}{2}$$

$$= 1860.$$

故选：A．

15.

解：(I) 设等比数列 $\{a_n\}$ 公比为 q ，由题意可得：

$$a_1 + a_2 = a_1(1 + q) = 6 \quad \text{①}$$

$$a_3 + a_4 = s_4 - s_2 = a_1 q^2 (1 + q) = 24 \quad \text{②} \dots\dots\dots 2 \text{ 分}$$

①、②联立,解得:

$$\begin{cases} a_1 = 2, \\ q = 2 \end{cases} \text{ 或 } \begin{cases} a_1 = -6 \\ q = -2 \end{cases} \text{ (舍)}$$

$$\therefore a_n = 2^n \dots\dots\dots 3 \text{ 分}$$

$$\because b_n \cdot b_{n+1} = a_n = 2^n, \quad \text{③}$$

\therefore 当 $n \geq 2$ 时,有

$$b_{n-1} \cdot b_n = 2^{n-1} \quad \text{④}$$

由 $\frac{\text{③}}{\text{④}}$ 可得:

$$\frac{b_{n+1}}{b_{n-1}} = 2 \quad (n \geq 2) \dots\dots\dots 5 \text{ 分}$$

又 $b_1 = 1$,

$\therefore b_2 = \frac{a_1}{b_1} = 2$, 从而 $b_1, b_3, \dots, b_{2n-1}$ 是首项为 $b_1 = 1$, 公比为 2 的等比数列,

b_2, b_4, \dots, b_{2n} 是前项为 $b_2 = 2$, 公比为 2 的等比数列, $\dots\dots\dots 6 \text{ 分}$

$$\therefore b_n = \begin{cases} 2^{\frac{n-1}{2}}, n \text{ 为奇数} \\ 2^{\frac{n}{2}}, n \text{ 为偶数} \end{cases} \dots\dots\dots 7 \text{ 分}$$

(Ⅱ) 当 n 为奇数时,

$$\begin{aligned} T_n &= (b_1 + b_3 + \dots + b_n) + (b_2 + b_4 + \dots + b_{n-1}) \\ &= \frac{1 \cdot (1 - 2^{\frac{n+1}{2}})}{1 - 2} + \frac{2 \cdot (1 - 2^{\frac{n-1}{2}})}{1 - 2} \\ &= 2^{\frac{n+1}{2}} - 1 + 2^{\frac{n-1}{2}} - 2 \\ &= 2^{\frac{n+3}{2}} - 3 \dots\dots\dots 9 \text{ 分} \end{aligned}$$

当 n 为偶数时,

$$\begin{aligned} T_n &= \frac{1 \cdot (1 - 2^{\frac{n}{2}})}{1 - 2} + \frac{2 \cdot (1 - 2^{\frac{n-1}{2}})}{1 - 2} \\ &= 2^{\frac{n}{2}} - 1 + 2 \cdot 2^{\frac{n-1}{2}} - 2 \\ &= 3 \cdot 2^{\frac{n-1}{2}} - 3 \dots\dots\dots 11 \text{ 分} \end{aligned}$$

\therefore 数列 $\{b_n\}$ 的前 n 项和为:

$$T_n = \begin{cases} 2^{\frac{n+3}{2}} - 3, n \text{ 为奇数} \\ 3 \cdot 2^{\frac{n-1}{2}} - 3, n \text{ 为偶数} \end{cases} \dots\dots\dots 12 \text{ 分}$$

16.证明: $\because a_n a_{n+1} = \lambda S_n - 1, a_{n+1} a_{n+2} = \lambda S_{n+1} - 1,$

$$\therefore a_{n+1} (a_{n+2} - a_n) = \lambda a_{n+1}$$

$$\because a_{n+1} \neq 0,$$

$$\therefore a_{n+2} - a_n = \lambda.$$

(II) 解: ①当 $\lambda=0$ 时, $a_n a_{n+1} = -1$, 假设 $\{a_n\}$ 为等差数列, 设公差为 d .

则 $a_{n+2} - a_n = 0, \therefore 2d = 0$, 解得 $d = 0$,

$$\therefore a_n = a_{n+1} = 1,$$

$\therefore 1^2 = -1$, 矛盾, 因此 $\lambda=0$ 时 $\{a_n\}$ 不为等差数列.

②当 $\lambda \neq 0$ 时, 假设存在 λ , 使得 $\{a_n\}$ 为等差数列, 设公差为 d .

$$\text{则 } \lambda = a_{n+2} - a_n = (a_{n+2} - a_{n+1}) + (a_{n+1} - a_n) = 2d,$$

$$\therefore d = \frac{\lambda}{2}.$$

$$\therefore a_n = 1 + \frac{\lambda(n-1)}{2}, \quad a_{n+1} = 1 + \frac{\lambda n}{2},$$

$$\therefore \lambda S_n = 1 + \left[1 + \frac{\lambda(n-1)}{2}\right] \left[1 + \frac{\lambda n}{2}\right] = \frac{\lambda^2}{4} n^2 + \left(\lambda - \frac{\lambda^2}{4}\right) n + 2 - \frac{\lambda}{2},$$

根据 $\{a_n\}$ 为等差数列的充要条件是
$$\begin{cases} \lambda \neq 0 \\ 2 - \frac{\lambda}{2} = 0 \end{cases}, \text{ 解得 } \lambda = 4.$$

此时可得 $S_n = n^2, a_n = 2n - 1$.

因此存在 $\lambda=4$, 使得 $\{a_n\}$ 为等差数列.