

一般数列求和专项靶题参考答案:

分组求和

1. 【详解】 (1) 由题意得 $\begin{cases} 2b_1 = a_2 + a_3, \\ 2b_2 = a_1 + a_2 \end{cases}$, 又 $a_2 = 3$,

设 $\{b_n\}$ 的公比为 q ,

故 $\begin{cases} 2b_1 = 3 + 3 + d \\ 2b_2 = 3 - d + 3 \end{cases}$, 相加得 $b_1 + b_2 = 6$, 则 $b_1 + b_1 q = 6$ ①,

两式相除得 $\frac{1}{q} = \frac{6+d}{6-d}$ ②,

又 $b_3 = a_1$, 所以 $b_1 q^2 = 3 - d$ ③,

由 ①③ 得 $\frac{1+q}{q^2} = \frac{6}{3-d}$ ④,

由 ②④ 得 $\frac{1 + \frac{6-d}{6+d}}{\left(\frac{6-d}{6+d}\right)^2} = \frac{6}{3-d}$, 解得 $(6-2d)(6+d) = (6-d)^2$,

解得 $d = 2$ 或 0 (舍去),

由 $\begin{cases} 2b_1 = 3 + 3 + d \\ 2b_2 = 3 - d + 3 \end{cases}$ 得, $\begin{cases} b_1 = 4 \\ b_2 = 2 \end{cases}$,

所以 $q = \frac{b_2}{b_1} = \frac{1}{2}$, 所以 $b_n = 4 \times \left(\frac{1}{2}\right)^{n-1} = 2^{3-n}$,

其中 $a_1 = 3 - d = 1$, 故 $a_n = 1 + 2(n-1) = 2n-1$,

(2) $a_n + b_n = 2n-1 + 2^{3-n}$,

其中 $1+3+5+\cdots+2n-1 = \frac{n(1+2n-1)}{2} = n^2$,

$$2^2 + 2 + 1 + \cdots + 2^{3-n} = \frac{4 - 2^{3-n} \cdot \frac{1}{2}}{1 - \frac{1}{2}} = 8 - 2^{3-n},$$

故 $S_n = n^2 + 8 - 2^{3-n}$

2. 【详解】 (1) 设数列 $\{a_n\}$ 的首项为 a_1 , 数列 $\{b_n\}$ 的公差为 d ,

因为 $a_2 = b_3, a_3 = b_{14}$,

可得 $3a_1 = 1 + 4d, 9a_1 = 1 + 13d \Leftrightarrow a_1 = 3, d = 2$,

所以 $a_n = 3^n, b_n = 2n-1$

(2) 由 (1) 知, $c_n = 3^n + (-1)^n (2n-1)$,

所以 $S_{2n} = 3^1 + 3^2 + \cdots + 3^{2n} + (-1) \times 1 + 1 \times 3 + (-1) \times 5 + 1 \times 7 + \cdots + (-1) \times (4n-3) + 1 \times (4n-1)$,

$$S_{2n} = \frac{3(1-3^{2n})}{1-3} + 2n = \frac{1}{2} \times 3^{2n+1} + 2n - \frac{3}{2}.$$

倒序相加

1. 【详解】 (1) 因为 $f(a) = \frac{4^a}{4^a+2}$, $f(1-a) = \frac{4^{1-a}}{4^{1-a}+2}$,

$$\text{所以, } f(a) + f(1-a) = \frac{4^a}{4^a+2} + \frac{4^{1-a}}{4^{1-a}+2} = \frac{4^a(4^{1-a}+2) + 4^{1-a}(4^a+2)}{(4^a+2)(4^{1-a}+2)} = \frac{8+2(4^a+4^{1-a})}{8+2(4^a+4^{1-a})} = 1.$$

$$(2) \text{ 由 (1) 可得, } f\left(\frac{1}{1001}\right) + f\left(\frac{1000}{1001}\right) = f\left(\frac{2}{1001}\right) + f\left(\frac{999}{1001}\right) = \cdots = f\left(\frac{1000}{1001}\right) + f\left(\frac{1}{1001}\right) = 1.$$

所以,

$$\begin{aligned} & f\left(\frac{1}{1001}\right) + f\left(\frac{2}{1001}\right) + f\left(\frac{3}{1001}\right) + \cdots + f\left(\frac{999}{1001}\right) + f\left(\frac{1000}{1001}\right) \\ & + f\left(\frac{1000}{1001}\right) + f\left(\frac{999}{1001}\right) + \cdots + f\left(\frac{2}{1001}\right) + f\left(\frac{1}{1001}\right) = \\ & f\left(\frac{1}{1001}\right) + f\left(\frac{1000}{1001}\right) + f\left(\frac{2}{1001}\right) + f\left(\frac{999}{1001}\right) + \cdots + f\left(\frac{1000}{1001}\right) + f\left(\frac{1}{1001}\right) = 1000, \end{aligned}$$

$$\text{所以 } f\left(\frac{1}{1001}\right) + f\left(\frac{2}{1001}\right) + f\left(\frac{3}{1001}\right) + \cdots + f\left(\frac{1000}{1001}\right) = 500.$$

故答案为: 1; 500.

$$2. \text{ 【详解】 因 } f(x) + f(-x) = \frac{2}{2^x+1} + \frac{2}{2^{-x}+1} = \frac{2(2^x+2^{-x}+2)}{2^x+2^{-x}+2} = 2,$$

设 $S = f(-5) + f(-4) + \cdots + f(0) + \cdots + f(4) + f(5)$, 则

$$2S = f(-5) + f(5) + f(-4) + f(4) + \cdots + 2f(0) + \cdots + f(4) + f(-4) + f(5) + f(-5) = 22, \text{ 故 } S = 11.$$

故答案为: 11

$$3. \text{ 【详解】 } f(1-x) + f(1+x) = (1-x)^3 - 3(1-x)^2 + (1+x)^3 - 3(1+x)^2 = -4,$$

$$\text{即 } f(x) + f(2-x) = -4$$

$$\text{设 } M = f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \cdots + f\left(\frac{4044}{2023}\right) + f\left(\frac{4045}{2023}\right) \quad \text{①},$$

$$\text{则 } M = f\left(\frac{4045}{2023}\right) + f\left(\frac{4044}{2023}\right) + \cdots + f\left(\frac{2}{2023}\right) + f\left(\frac{1}{2023}\right) \quad \text{②}$$

①+②得

$$\begin{aligned} 2M &= \left[f\left(\frac{1}{2023}\right) + f\left(\frac{4045}{2023}\right) \right] + \left[f\left(\frac{2}{2023}\right) + f\left(\frac{4044}{2023}\right) \right] + \cdots + \left[f\left(\frac{4044}{2023}\right) + f\left(\frac{2}{2023}\right) \right] + \left[f\left(\frac{4045}{2023}\right) + f\left(\frac{1}{2023}\right) \right] \\ &= -4 \times 4045, \end{aligned}$$

所以 $M = -8090$,

$$\text{又 } f\left(\frac{4046}{2023}\right) = f(2) = 8 - 12 = -4,$$

$$\text{所以 } f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \cdots + f\left(\frac{4046}{2023}\right) = -8090 - 4 = -8094.$$

故选: D.

$$\mathbf{4.【详解】}$$
 由于函数 $f\left(x + \frac{1}{2}\right)$ 为奇函数, 则 $f\left(-x + \frac{1}{2}\right) = -f\left(x + \frac{1}{2}\right)$,

$$\text{即 } f\left(\frac{1}{2} - x\right) + f\left(\frac{1}{2} + x\right) = 0, \text{ 所以 } f(x) + f(1-x) = 0,$$

$$\text{所以 } g(x) + g(1-x) = [f(x) + 1] + [f(1-x) + 1] = 2,$$

$$\begin{aligned} \text{所以 } 2(a_1 + a_2 + \cdots + a_{2022}) &= 2\left[g\left(\frac{1}{2023}\right) + g\left(\frac{2}{2023}\right) + \cdots + g\left(\frac{2022}{2023}\right)\right] \\ &= \left[g\left(\frac{1}{2023}\right) + g\left(\frac{2022}{2023}\right)\right] + \left[g\left(\frac{2}{2023}\right) + g\left(\frac{2021}{2023}\right)\right] + \cdots + \left[g\left(\frac{2022}{2023}\right) + g\left(\frac{1}{2023}\right)\right] = 2 \times 2022, \end{aligned}$$

因此数列 $\{a_n\}$ 的前 2022 项和为 $a_1 + a_2 + \cdots + a_{2022} = 2022$,

故答案为: 2022

$$\mathbf{5.【详解】}$$
 (1) 因为 $\{a_n\}$ 是等比数列, 公比为 $q \neq -1$, 则 $a_4 = a_1 q^3$, $a_5 = a_1 q^4$, $a_7 = a_1 q^6$, $a_8 = a_1 q^7$,

$$\text{所以 } \frac{a_4 + a_5}{a_7 + a_8} = \frac{a_1 q^3 + a_1 q^4}{a_1 q^6 + a_1 q^7} = \frac{1}{q^3} = \frac{1}{27}, \text{ 解得 } q = 3,$$

$$\text{由 } S_4 = a_3 + 93, \text{ 可得 } \frac{a_1(1-3^4)}{1-3} = 9a_1 + 93, \text{ 解得 } a_1 = 3,$$

所以数列 $\{a_n\}$ 的通项公式为 $a_n = 3^n$.

$$(2) \text{ 由 (1) 得 } b_n = \begin{cases} -n, & n \text{ 为奇数} \\ 3^n, & n \text{ 为偶数} \end{cases},$$

$$\text{当 } n \text{ 为偶数时, } T_n = b_1 + b_2 + \cdots + b_n = (b_1 + b_3 + \cdots + b_{n-1}) + (b_2 + b_4 + \cdots + b_n)$$

$$= -(1+3+\cdots+n-1) + (3^2+3^4+\cdots+3^n) = -\frac{\frac{n}{2} \cdot [1+(n-1)]}{2} \times + \frac{9\left(1-9^{\frac{n}{2}}\right)}{1-9}$$

$$= \frac{9}{8}(3^n - 1) - \frac{n^2}{4};$$

$$\text{当 } n \text{ 为奇数时 } T_n = T_{n+1} - b_{n+1} = \frac{9}{8}(3^{n+1} - 1) - \frac{(n+1)^2}{4} \cdot 3^{n+1} = \frac{1}{8} \times 3^{n+1} - \frac{9}{8} - \frac{(n+1)^2}{4};$$

$$\text{综上所述: } T_n = \begin{cases} \frac{1}{8} \times 3^{n+1} - \frac{9}{8} - \frac{(n+1)^2}{4}, & n \text{ 为奇数} \\ \frac{9}{8}(3^n - 1) - \frac{n^2}{4}, & n \text{ 为偶数} \end{cases}.$$

6. 【详解】 (1) 设数列 $\{a_n\}$ 的公差为 d , 则 $d \neq 0$,

又 $a_1 = 1$, 所以 $a_n = 1 + (n-1)d$

因为 a_1, a_2, a_6 成等比数列,

所以 $(1+d)^2 = 1+5d$,

化简得 $d^2 - 3d = 0$, 又 $d \neq 0$,

所以 $d = 3$,

所以 $a_n = 1 + 3(n-1) = 3n - 2$;

(2) 由 (1) 可得: $b_n = (-1)^n(3n-2)$,

则 $b_{2k-1} + b_{2k} = (-1)^{2k-1}(6k-5) + (-1)^{2k}(6k-2) = 3$,

则当 n 为偶数时, $S_n = 3 \times \frac{n}{2} = \frac{3n}{2}$,

当 n 为奇数时, $S_n = S_{n-1} + b_n = \frac{3(n-1)}{2} - (3n-2) = \frac{1-3n}{2}$,

$$\text{即 } S_n = \begin{cases} \frac{3n}{2}, & n \text{ 为偶数} \\ \frac{1-3n}{2}, & n \text{ 为奇数} \end{cases}.$$

裂项相消法

1. 【详解】 (1) 设数列 $\{a_n\}$ 的公差为 d , 数列 $\{b_n\}$ 的等比为 q ,

因为 $a_2 + b_2 = 4$, $S_3 = 6$, $a_1 = b_1 = 1$,

所以 $\begin{cases} 1+d+q=4 \\ 3+3d=6 \end{cases}$, 解得 $\begin{cases} d=1 \\ q=2 \end{cases}$

$a_n = 1 + (n-1) \times 1 = n$, $b_n = 2^{n-1}$.

(2) 因为 $a_n = n$,

所以 $S_n = \frac{(a_1 + a_n)n}{2} = \frac{n(1+n)}{2}$,

则 $\frac{1}{S_n} = \frac{2}{n(1+n)} = 2\left(\frac{1}{n} - \frac{1}{n+1}\right)$,

所以 $T_n = \frac{1}{S_1} + \frac{1}{S_2} + \cdots + \frac{1}{S_{n-1}} + \frac{1}{S_n}$

$$= 2 \left(\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \cdots + \frac{1}{n-1} - \frac{1}{n} + \frac{1}{n} - \frac{1}{n+1} \right)$$

$$= 2 \left(1 - \frac{1}{n+1} \right) = 2 - \frac{2}{n+1} = \frac{2n}{n+1}.$$

2. 【详解】 (1) 设等差数列 $\{a_n\}$ 的公差为 d ，等比数列 $\{b_n\}$ 的公比为 q ，

$$\text{则} \begin{cases} a_2 = a_1 + d = 4 \\ 2a_4 - a_5 = 2(a_1 + 3d) - (a_1 + 4d) = 7 \end{cases}, \text{解得 } a_1 = 1, d = 3, \text{ 则 } a_n = 3n - 2;$$

$$\begin{cases} b_3 = b_1 q^2 = 4 \\ b_4 + b_5 = q^3(b_1 + b_2) = 8(b_1 + b_2) \end{cases}, \text{ 由于 } q \neq -1, \text{ 则 } b_1 + b_2 = b_1(1+q) \neq 0,$$

故解得 $b_1 = 1, q = 2$ ，则 $b_n = 2^{n-1}$.

$$(2) \quad c_n = \frac{3}{a_n \cdot a_{n+1}} = \frac{3}{(3n-2)(3n+1)} = \frac{1}{3n-2} - \frac{1}{3n+1},$$

$$\text{所以 } S_n = 1 - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \cdots + \frac{1}{3n-2} - \frac{1}{3n+1} = 1 - \frac{1}{3n+1} = \frac{3n}{3n+1}.$$

$$\mathbf{3. 【详解】} (1) \text{ 解法一: 由 } a_{n+1} = \sqrt{\frac{n+1}{n}} a_n \text{ 得 } \frac{a_{n+1}}{a_n} = \sqrt{\frac{n+1}{n}} = \frac{\sqrt{n+1}}{\sqrt{n}},$$

$$\text{由累乘法得 } a_n = a_1 \cdot \frac{a_2}{a_1} \cdot \frac{a_3}{a_2} \cdots \frac{a_n}{a_{n-1}} = a_1 \cdot \frac{\sqrt{2}}{1} \times \frac{\sqrt{3}}{\sqrt{2}} \times \cdots \times \frac{\sqrt{n}}{\sqrt{n-1}} = \sqrt{n}.$$

$$\text{解法二: 由 } a_{n+1} = \sqrt{\frac{n+1}{n}} a_n \text{ 得 } \frac{a_{n+1}}{\sqrt{n+1}} = \frac{a_n}{\sqrt{n}},$$

则数列 $\left\{ \frac{a_n}{\sqrt{n}} \right\}$ 是各项为 1 的常数列，所以 $\frac{a_n}{\sqrt{n}} = 1$ ，即 $a_n = \sqrt{n}$.

$$(2) \text{ 由 (1) 得 } \frac{1}{a_n + a_{n+1}} = \frac{1}{\sqrt{n} + \sqrt{n+1}} = \sqrt{n+1} - \sqrt{n},$$

$$\text{所以 } S_n = (\sqrt{2} - 1) + (\sqrt{3} - \sqrt{2}) + \cdots + (\sqrt{n+1} - \sqrt{n}) = \sqrt{n+1} - 1.$$

4. 【详解】 (1) 令 $n=1$ ， $S_1^2 - (1+1-3)S_1 - 3(1+1) = 0$ ，则 $a_1 = -3$ 舍去，

所以 $a_1 = 2$.

$$(2) \because S_n^2 - (n^2 + n - 3)S_n - 3(n^2 + n) = 0, \therefore (S_n + 3)(S_n - n^2 - n) = 0,$$

因为数列 $\{a_n\}$ 各项均为正数， $S_n \neq -3$ 舍去，

$$\therefore S_n = n^2 + n, \text{ 当 } n \geq 2 \text{ 时},$$

$$\therefore S_{n-1} = (n-1)^2 + (n-1), \therefore a_n = S_n - S_{n-1} = 2n,$$

$$\therefore a_n = \begin{cases} 2, n=1 \\ S_n - S_{n-1} = 2n, n \geq 2 \end{cases} \therefore a_n = 2n$$

$$\begin{aligned}
 (3) \text{ 令 } b_n &= \frac{1}{2n\sqrt{2n+2}} = \frac{1}{2\sqrt{2n}\sqrt{n+1}} = \frac{1}{\sqrt{2n}(\sqrt{n+1} + \sqrt{n+1})} \\
 &\leq \frac{1}{\sqrt{2}\sqrt{n^2-1}(\sqrt{n+1} + \sqrt{n-1})} = \frac{1}{\sqrt{2}\sqrt{(n-1)(n+1)}(\sqrt{n+1} + \sqrt{n-1})} \\
 &= \frac{\sqrt{n+1} - \sqrt{n-1}}{2\sqrt{2}\sqrt{(n-1)(n+1)}} = \frac{\sqrt{2}}{4} \left(\frac{1}{\sqrt{n-1}} - \frac{1}{\sqrt{n+1}} \right) (n \geq 2),
 \end{aligned}$$

$$\begin{aligned}
 \text{所以 } S_n &= b_1 + b_2 + \cdots + b_n \leq b_1 + \frac{\sqrt{2}}{4} \left(1 - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{n-1}} - \frac{1}{\sqrt{n+1}} \right) \\
 &= \frac{1}{4} + \frac{\sqrt{2}}{4} \left(1 + \frac{\sqrt{2}}{2} - \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) = \frac{2 + \sqrt{2}}{4} - \frac{\sqrt{2}}{4} \left(\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} \right)
 \end{aligned}$$

5. 【详解】 (1) 设 $\{a_n\}$ 的公比为 $q > 0$,

因为 $a_3 = a_2 + 2$, 所以 $a_1 q^2 = a_1 q + 2$, 即 $q^2 - q - 2 = 0$, 解得 $q = 2$ 或 $q = -1$ (舍),

所以 $a_n = a_1 q^{n-1} = 2^{n-1}$,

设 $\{b_n\}$ 的公差为 d ,

因为 $a_4 = b_3 + b_5$, $a_5 = b_4 + 2b_6$, 所以 $b_3 + b_5 = 8$, $b_4 + 2b_6 = 16$,

所以 $\begin{cases} 2b_1 + 6d = 8 \\ 3b_1 + 13d = 16 \end{cases}$, 解得 $d = b_1 = 1$, 所以 $b_n = b_1 + (n-1)d = n$.

$$(2) \quad c_n = \frac{a_n}{(a_n + 1)(a_{n+1} + 1)} = \frac{2^{n-1}}{(2^{n-1} + 1)(2^n + 1)} = \frac{1}{2^{n-1} + 1} - \frac{1}{2^n + 1},$$

$$\begin{aligned}
 \text{所以 } T_n &= c_1 + c_2 + c_3 + \cdots + c_n = \frac{1}{2^0 + 1} - \frac{1}{2^1 + 1} + \frac{1}{2^1 + 1} - \frac{1}{2^2 + 1} + \frac{1}{2^2 + 1} - \frac{1}{2^3 + 1} + \cdots + \frac{1}{2^{n-1} + 1} - \frac{1}{2^n + 1} \\
 &= \frac{1}{2} - \frac{1}{2^n + 1}.
 \end{aligned}$$

6 【详解】 (1) 因为 $2S_n = (3^n - 1)a_n$,

所以, 当 $n \geq 2$ 时, $2S_{n-1} = (3^{n-1} - 1)a_{n-1}$,

两式相减得, $2S_n - 2S_{n-1} = (3^n - 1)a_n - (3^{n-1} - 1)a_{n-1}$,

化简可得 $(3^n - 3)a_n - (3^{n-1} - 1)a_{n-1} = 0$,

所以 $3a_n - a_{n-1} = 0$, 即 $3a_n = a_{n-1}$,

又 $a_1 = \frac{1}{3} \neq 0$, 可得 $\frac{a_n}{a_{n-1}} = \frac{1}{3}$,

所以数列 $\{a_n\}$ 是以 $\frac{1}{3}$ 为首项, $\frac{1}{3}$ 为公比的等比数列,

$$\text{可得 } a_n = \frac{1}{3} \cdot \left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^n$$

$$(2) \text{ 由 (1) 可知, } a_{n+1} = \left(\frac{1}{3}\right)^{n+1},$$

$$S_n = \frac{\frac{1}{3} \left[1 - \left(\frac{1}{3}\right)^n \right]}{1 - \frac{1}{3}} = \frac{1}{2} \left[1 - \left(\frac{1}{3}\right)^n \right]$$

$$\text{所以 } S_{n+1} = \frac{1}{2} \left[1 - \left(\frac{1}{3}\right)^{n+1} \right],$$

$$\text{则 } b_n = \frac{a_{n+1}}{(1-a_n)S_{n+1}} = \frac{\left(\frac{1}{3}\right)^{n+1}}{\left(1 - \left(\frac{1}{3}\right)^n\right) \times \frac{1}{2} \left[1 - \left(\frac{1}{3}\right)^{n+1} \right]},$$

$$= \frac{1}{1 - \left(\frac{1}{3}\right)^n} - \frac{1}{1 - \left(\frac{1}{3}\right)^{n+1}},$$

$$T_n = b_1 + b_2 + b_3 + \cdots + b_n$$

$$= \frac{1}{1 - \left(\frac{1}{3}\right)^1} - \frac{1}{1 - \left(\frac{1}{3}\right)^2} + \frac{1}{1 - \left(\frac{1}{3}\right)^2} - \frac{1}{1 - \left(\frac{1}{3}\right)^3} + \cdots + \frac{1}{1 - \left(\frac{1}{3}\right)^n} - \frac{1}{1 - \left(\frac{1}{3}\right)^{n+1}},$$

$$= \frac{3}{2} - \frac{1}{1 - \left(\frac{1}{3}\right)^{n+1}},$$

$$\text{因为 } 1 - \left(\frac{1}{3}\right)^{n+1} \geq \frac{8}{9}, \text{ 所以 } \frac{1}{1 - \left(\frac{1}{3}\right)^{n+1}} \leq \frac{9}{8},$$

$$\text{则 } T_n \geq \frac{3}{8}.$$

错位相减法

$$1. \text{【详解】 (1) 设等差数列 } \{a_n\} \text{ 的公差为 } d, \text{ 由题意可知 } \begin{cases} a_1 + d = 3 \\ 5a_1 + \frac{5 \times 4}{2} \times d = 20 \end{cases},$$

$$\text{解得 } \begin{cases} a_1 = 2 \\ d = 1 \end{cases}, \therefore a_n = n + 1,$$

$$\text{由 } T_n = 1 - b_n, \text{ 则当 } n \geq 2 \text{ 时, 有 } T_{n-1} = 1 - b_{n-1},$$

$$\text{则 } T_n - T_{n-1} = 1 - b_n - (1 - b_{n-1}) = b_{n-1} - b_n = b_n,$$

$$\text{故 } b_n = \frac{1}{2} b_{n-1}, \text{ 当 } n=1 \text{ 时, 有 } T_1 = 1 - b_1 = b_1, \text{ 故 } b_1 = \frac{1}{2},$$

即数列 $\{b_n\}$ 是以 $\frac{1}{2}$ 为首项, $\frac{1}{2}$ 为公比的等比数列,

$$\therefore b_n = \frac{1}{2^n};$$

$$(2) \text{ 由 (1) 知 } a_n \cdot b_n = (n+1) \frac{1}{2^n}, n \in \mathbf{N}^*,$$

$$\text{故 } R_n = 2 \times \frac{1}{2} + 3 \times \frac{1}{2^2} + \cdots + (n+1) \frac{1}{2^n},$$

$$\text{则 } \frac{1}{2} R_n = 2 \times \frac{1}{2^2} + \cdots + n \cdot \frac{1}{2^n} + (n+1) \frac{1}{2^{n+1}},$$

$$\text{则 } R_n - \frac{1}{2} R_n = \frac{1}{2} R_n = 1 + \frac{1}{2^2} + \cdots + \frac{1}{2^n} - \frac{n+1}{2^{n+1}}$$

$$= 1 + \frac{\frac{1}{4} \left(1 - \frac{1}{2^{n+1}} \right)}{1 - \frac{1}{2}} - \frac{n+1}{2^{n+1}} = \frac{3}{2} - \frac{1}{2^n} - \frac{n+1}{2^{n+1}},$$

$$\therefore R_n = 3 - \frac{n+3}{2^n}.$$

$$2. \text{ 【详解】 (1) 当 } n=1 \text{ 时, } a_1^3 = \left(\frac{1+1}{2} \right)^2 = 1, \text{ 所以 } a_1 = 1,$$

$$\text{因为 } a_1^3 + a_2^3 + \cdots + a_n^3 = \left(\frac{n^2 + n}{2} \right)^2 \quad ①$$

$$\text{当 } n > 1 \text{ 时, } a_1^3 + a_2^3 + \cdots + a_{n-1}^3 = \left[\frac{(n-1)^2 + (n-1)}{2} \right]^2 \quad ②,$$

$$\text{将 } ① - ② \text{ 得 } a_n^3 = \left(\frac{n^2 + n}{2} \right)^2 - \left[\frac{(n-1)^2 + (n-1)}{2} \right]^2 = n^3,$$

当 $n=1$ 时也适用, 所以 $a_n = n$,

所以数列 $\{a_n\}$ 的通项公式为 $a_n = n$.

$$(2) \text{ 由 } b_n = \frac{a_n}{3^n} = \frac{n}{3^n} = n \cdot \left(\frac{1}{3} \right)^n,$$

$$\text{所以 } S_n = 1 \cdot \left(\frac{1}{3} \right)^1 + 2 \cdot \left(\frac{1}{3} \right)^2 + 3 \cdot \left(\frac{1}{3} \right)^3 + \cdots + (n-1) \left(\frac{1}{3} \right)^{n-1} + n \cdot \left(\frac{1}{3} \right)^n \quad ③,$$

$$\frac{1}{3} S_n = 1 \cdot \left(\frac{1}{3} \right)^2 + 2 \cdot \left(\frac{1}{3} \right)^3 + \cdots + (n-2) \cdot \left(\frac{1}{3} \right)^{n-1} + (n-1) \cdot \left(\frac{1}{3} \right)^n + n \cdot \left(\frac{1}{3} \right)^{n+1} \quad ④,$$

将③-④得 $\frac{2}{3}S_n = \left(\frac{1}{3}\right)^1 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \cdots + \left(\frac{1}{3}\right)^{n-1} + \left(\frac{1}{3}\right)^n - n \cdot \left(\frac{1}{3}\right)^{n+1}$,

化简得 $S_n = \frac{3}{4} - \left(\frac{1}{3}\right)^n \cdot \left(\frac{3}{4} + \frac{n}{2}\right)$.