

等差、等比数列专题参考答案:

1 答案 D

解析 $\because a_m - a_n = (m - n)d = n - m, \therefore d = -1, \therefore a_{m+n} = a_m + nd = n - n = 0.$

2. 答案 C

解析 $\because S_{m-1} = -2, S_m = 0, S_{m+1} = 3,$

$\therefore a_m = S_m - S_{m-1} = 0 - (-2) = 2, a_{m+1} = S_{m+1} - S_m = 3 - 0 = 3.$

$\therefore d = a_{m+1} - a_m = 3 - 2 = 1.$

$\because S_m = ma_1 + \frac{m(m-1)}{2} \times 1 = 0, \therefore a_1 = -\frac{m-1}{2}.$

又 $\because a_{m+1} = a_1 + m \times 1 = 3, \therefore -\frac{m-1}{2} + m = 3.$

$\therefore m = 5.$ 故选 C.

3. [答案] B

[解析] $\because \{a_n\}$ 是等差数列,

$$\therefore \begin{cases} 3a_3 = 105 \\ 3a_4 = 99 \end{cases} \Rightarrow \begin{cases} a_3 = 35 \\ a_4 = 33 \end{cases} \Rightarrow \begin{cases} a_1 = 39 \\ d = -2 \end{cases},$$

$\therefore a_{20} = a_1 + 19d = 1,$ 故选 B.

4. [答案] C

[解析] $\because a_{m-1} + a_{m+1} - a_m^2 = 0,$

$\therefore 2a_m - a_m^2 = 0, \therefore a_m = 0$ 或 $a_m = 2.$

又 $\because S_{2m-1} = 38, \therefore \frac{(2m-1)(a_1 + a_{2m-1})}{2} = 38,$

$(2m-1) \cdot a_m = 38, \therefore a_m = 2,$

$\therefore 2(2m-1) = 38, \therefore 2m-1 = 19, \therefore m = 10.$

5. 210 [解析] 在等差数列中, $S_m, S_{2m} - S_m, S_{3m} - S_{2m}$ 成等差数列.

$\therefore 30, 70, S_{3m} - 100$ 成等差数列.

$\therefore 2 \times 70 = 30 + (S_{3m} - 100), \therefore S_{3m} = 210.$

6. [答案] A

[解析] 据等差数列前 n 项和性质可知: $S_3, S_6 - S_3, S_9 - S_6, S_{12} - S_9$ 仍成等差数列.

设 $S_3 = k$, 则 $S_6 = 3k, S_6 - S_3 = 2k,$

$\therefore S_9 - S_6 = 3k, S_{12} - S_9 = 4k,$

$\therefore S_9 = S_6 + 3k = 6k, S_{12} = S_9 + 4k = 10k,$

$$\therefore \frac{S_6}{S_{12}} = \frac{3k}{10k} = \frac{3}{10}.$$

7. 答案 A

解析 由题意可得 $\frac{S_4}{S_5} = \frac{\frac{4(a_1+a_4)}{2}}{\frac{5(a_1+a_5)}{2}} = \frac{2(a_2+a_3)}{5a_3} = \frac{8}{15}$.

8.答案 D

解析 $\frac{a_5}{b_5} = \frac{2a_5}{2b_5} = \frac{a_1+a_9}{b_1+b_9} = \frac{\frac{9}{2}(a_1+a_9)}{\frac{9}{2}(b_1+b_9)} = \frac{S_9}{T_9} = \frac{21}{4}$.

9.答案 D

解析 由关系式易知 $\left\{\frac{1}{x_n}\right\}$ 为首项为 $\frac{1}{x_1}=1$, $d=\frac{1}{2}$ 的等差数列, $\frac{1}{x_n}=\frac{n+1}{2}$, 所以 $x_n=\frac{2}{n+1}$.

10. [答案] C

[解析] 设 $x^2-2x+m=0$ 的根为 x_1, x_2 且 $x_1<x_2$, $x^2-2x+n=0$ 的根为 x_3, x_4 且 $x_3<x_4$, 且 $x_1=\frac{1}{4}$,

又 $x_1+x_2=2$, $\therefore x_2=\frac{7}{4}$,

又 $x_3+x_4=2$, 且 x_1, x_3, x_4, x_2 成等差数列,

\therefore 公差 $d=\frac{1}{3}\left(\frac{7}{4}-\frac{1}{4}\right)=\frac{1}{2}$, $\therefore x_3=\frac{3}{4}$, $x_4=\frac{5}{4}$.

$\therefore |m-n|=\left|\frac{1}{4}\times\frac{7}{4}-\frac{3}{4}\times\frac{5}{4}\right|=\frac{1}{2}$, 故选 C.

11. [答案] 169

[分析] 利用前 n 项和公式和二次函数性质求解.

[解析] 方法 1: 由 $S_{17}=S_9$, 得

$$25\times 17+\frac{17}{2}(17-1)d=25\times 9+\frac{9}{2}(9-1)d,$$

解得 $d=-2$,

$$\therefore S_n=25n+\frac{n}{2}(n-1)\cdot(-2)=-(n-13)^2+169,$$

\therefore 由二次函数性质, 当 $n=13$ 时, S_n 有最大值 169.

方法 2: 先求出 $d=-2$, $\therefore a_1=25>0$,

$$\text{由 } \begin{cases} a_n=25-2(n-1)\geq 0 \\ a_{n+1}=25-2n\leq 0 \end{cases}, \text{ 得 } \begin{cases} n\leq 13\frac{1}{2} \\ n\geq 12\frac{1}{2} \end{cases},$$

\therefore 当 $n=13$ 时, S_n 有最大值 169.

方法 3: 由 $S_{17}=S_9$ 得 $a_{10}+a_{11}+\cdots+a_{17}=0$,

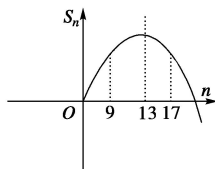
而 $a_{10}+a_{17}=a_{11}+a_{16}=a_{12}+a_{15}=a_{13}+a_{14}$,

故 $a_{13}+a_{14}=0$.

$\therefore d=-2<0$, $a_1>0$, $\therefore a_{13}>0$, $a_{14}<0$,

故 $n=13$ 时, S_n 有最大值.

方法 4: 由 $d=-2$ 得 S_n 的图象如图所示(图象上一些孤立点),



由 $S_{17}=S_9$ 知图象对称轴为 $n=\frac{9+17}{2}=13$,

\therefore 当 $n=13$ 时, S_n 取得最大值 169.

12. A [解析] 由 $S_{13}=\frac{13(a_1+a_{13})}{2}=\frac{13\cdot 2a_7}{2}=13a_7=26$, 得 $a_7>0$,

由 $S_{14}=\frac{14(a_1+a_{14})}{2}=\frac{14(a_7+a_8)}{2}=7(a_7+a_8)=-14$, 得 $a_8<-a_7<0$,

当 $n=7$ 时, S_n 取最大值, 故选 A.

13. 10 [解析] 在等差数列 $\{a_n\}$ 中, 所有奇数项的和 $S_{\text{奇}}=\frac{(n+1)(a_1+a_{2n+1})}{2}=165$,

所有偶数项的和为 $S_{\text{偶}}=\frac{n(a_2+a_{2n})}{2}=150$. $\because a_1+a_{2n+1}=a_2+a_{2n}$, $\therefore \frac{n+1}{n}=\frac{165}{150}=\frac{11}{10}$, $\therefore n=$

10.

16. [解析] (1) 当 $n\geq 2$ 时, $a_n=S_n-S_{n-1}$

$$=(100n-n^2)-[100(n-1)-(n-1)^2]$$

$$=101-2n.$$

$\because a_1=S_1=100\times 1-1^2=99$, 适合上式,

$$\therefore a_n=101-2n(n\in\mathbb{N}^*).$$

又 $a_{n+1}-a_n=-2$ 为常数,

\therefore 数列 $\{a_n\}$ 是首项为 99, 公差为 -2 的等差数列.

(2) 令 $a_n=101-2n\geq 0$, 得 $n\leq 50.5$,

$\because n\in\mathbb{N}^*$, $\therefore n\leq 50(n\in\mathbb{N}^*)$.

① 当 $1\leq n\leq 50$ 时, $a_n>0$, 此时 $b_n=|a_n|=a_n$,

$\therefore \{b_n\}$ 的前 n 项和 $S'_n=100n-n^2$.

② 当 $n\geq 51$ 时, $a_n<0$, 此时 $b_n=|a_n|=-a_n$,

$$\text{由 } b_{51}+b_{52}+\cdots+b_n=-(a_{51}+a_{52}+\cdots+a_n)$$

$$=-(S_n-S_{50})=S_{50}-S_n,$$

得数列 $\{b_n\}$ 的前 n 项和

$$S'_n=S_{50}+(S_{50}-S_n)=2S_{50}-S_n$$

$$=2\times 2500-(100n-n^2)=5000-100n+n^2.$$

由①、②得数列 $\{b_n\}$ 的前 n 项和为

$$S'_n=\begin{cases} 100n-n^2 & (n\in\mathbb{N}^*, 1\leq n\leq 50) \\ 5000-100n+n^2 & (n\in\mathbb{N}^*, n\geq 51) \end{cases}$$

等比数列类参考答案:

1 答案 2 或 3

解析 由数列 $\{c_{n+1}-pc_n\}$ 为等比数列, 得 $(c_3-pc_2)^2=(c_2-pc_1)(c_4-pc_3)$, 即 $(35-13p)^2=(13-5p)(97-35p)$. 解得 $p=2$ 或 $p=3$.

2. [答案] C

[解析] 解法 1: $\{a_n\}$ 为等比数列的充要条件是 $S_n=\frac{a_1}{1-q}(1-q^n)$, 由 $S_n=3^n+k$ 知 $k=-1$,

故选 C.

解法 2: $\because \frac{a_{n+1}}{a_n} = c \neq 0$, $\therefore \{a_n\}$ 为等比数列,

$$a_1 = S_1 = 3 + k, \quad a_2 = S_2 - S_1 = (9 + k) - (3 + k) = 6, \quad a_3 = S_3 - S_2 = 18,$$

$$\therefore \text{公比 } q = \frac{a_3}{a_2} = \frac{a_2}{a_1}, \quad \therefore \frac{18}{6} = \frac{6}{3+k}, \quad \therefore k = -1.$$

3. 解析 因为 $\{a_n\}$ 为等比数列,

$$\text{所以由已知可得 } a_{10}a_{11} = a_9a_{12} = a_1a_{20} = e^5.$$

$$\text{于是 } \ln a_1 + \ln a_2 + \cdots + \ln a_{20} = \ln(a_1a_2a_3 \cdots a_{20}).$$

$$\text{而 } a_1a_2a_3 \cdots a_{20} = (a_1a_{20})^{10} = (e^5)^{10} = e^{50},$$

$$\text{因此 } \ln a_1 + \ln a_2 + \cdots + \ln a_{20} = \ln e^{50} = 50.$$

4 答案 C

解析 由已知得, $S_m - S_{m-1} = a_m = -16$, $S_{m+1} - S_m = a_{m+1} = 32$, 故公比 $q = \frac{a_{m+1}}{a_m} = -2$. 又

$$S_m = \frac{a_1 - a_m q}{1 - q} = -11, \text{ 故 } a_1 = -1. \text{ 又 } a_m = a_1 \cdot q^{m-1} = -16, \text{ 故 } (-1) \times (-2)^{m-1} = -16, \text{ 求得 } m =$$

5.

5. $2^{n+1} - n - 2$ [解析] \because 数列的各项均可以看作是首项为 1, 公比为 2 的等比数列, 故

$$\text{此数列的通项公式为 } a_n = \frac{1 \times (2^n - 1)}{2 - 1} = 2^n - 1 (n \geq 1),$$

$$\therefore S_n = (2^1 - 1) + (2^2 - 1) + (2^3 - 1) + \cdots + (2^n - 1)$$

$$= 2 + 2^2 + 2^3 + \cdots + 2^n - n$$

$$= \frac{2 \times (2^n - 1)}{2 - 1} - n$$

$$= 2^{n+1} - n - 2.$$